A NEW MULTIPLIER USING DEAD-BAND TYPE NONLINEARITY

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ABSTRACT. An electronic analogue multiplier using dead-band type nonlinearity has been described. The multiplier can be readily set up by using standard operational amplifiers and a triangular wave generator. The static and dynamic accuracies of the multiplier are very high.

INTRODUCTION

In analogue computers the product of two machine variables X and Y is obtained using mainly two different types of electronic multipliers. These are either time-division or quarter-square multipliers. In the latter case mostly diode function generators are used for approximating the square-law characteristics.

The static accuracy in a multiplier using diode square law function generator is limited by the total number of straight line segments approximating the square characteristics in any quadrant. In order to increase the accuracy the number of straight line segments have to be increased, with a consequent increase in the number of diodes. The upper limit in static accuracy in such multipliers can be obtained only after careful adjustment of the circuit.

The static and dynamic accuracies of a time-division multiplier are comparable to those of other linear computing elements used in an electronic analogue computer. The performance of such multipliers has been much improved by using the excellent switching properties of transistors (E. Kettel and W. Schneider, 1961; W. R. Seegmiller, 1962). These features have made time-division multiplier very popular.

In the present paper, we shall describe a multiplier in which basically the principle of Quarter-square multiplication is used. In the circuit described, the squaring of the input signals X and Y is obtained by feeding a symmetrical triangular wave of high repetition frequency along with the signals X and Y into a symmetrical dead-band type nonlinearity. (Norsworthy, K. H., 1954; Mayer, R. A., Davis, H. B., 1956; Bengt, Jiewertz, 1958; Gomperts, R. J., Righton, D. W., Readshaw, D., 1957; and Philbrick Triangular Wave Multiplier). The multiplier has the advantage of extreme simplicity in its circuit arrangement and it overcomes many of the major disadvantages associated with diode multipliers using quarter-square principle. The only additional equipment required, besides the other computing elements available in an analogue computer, is a symmetrical triangular wave generator of high repetition frequency.

OPERATING PRINCIPLES OF MULTIPLIER

The transmission characteristics of a dead-band type nonlinearity is shown in Fig. I.,

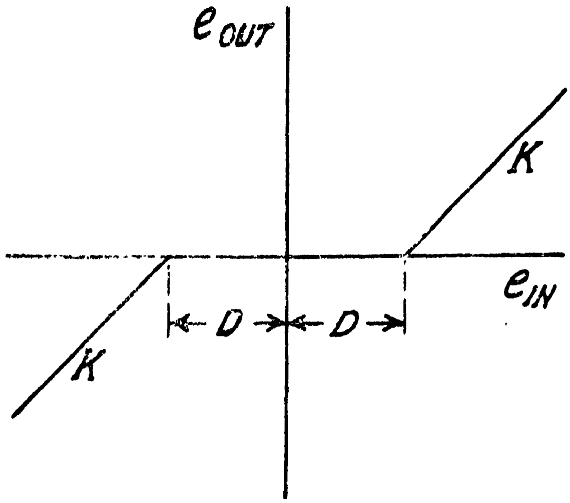

Fig. 1. Transmission characteristics of dead-band type nonlinearity.

$$\begin{aligned} e_{out} &= 0 & -D < e_{in} < D \\ &= K(e_{in}-D) & D < e_{in} \\ &= K(e_{in}+D) & -D > e_{in} \end{aligned}$$

where D is the dead-band width,

K is the slope of the dead-band.

If the input is a symmetrical triangular wave of amplitude $B(< D)$ mixed with a d.c. singal X, and the outputs from the positive and negative halves of the dead-band element are added, then the average value of the output is given by

$$V_1^{*} = K\left(1-\frac{D}{B}\right)X \tag{1}$$

If the dead-band width is increased or decreased symmetrically by another d.c. signal Y, the average value of the output from the positive half of the non-linearity will be given by

$$V_{p} = \frac{K}{4}\left[\, B-2(D\pm Y-X)+\frac{(D\pm Y-X)^2}{B}\,\right] \quad \dots \tag{2}$$

and that from the negative half by

$$V_N = \frac{K}{4}\left[B-2(D\pm Y+X)+\frac{(D\pm Y+X)^2}{B}\right] \qquad \ldots \quad (3)$$

and the average value of the added output is given by

$$V_2 = \left(1-\frac{D\pm Y}{B}\right)KX$$

$$= V_1 \mp \frac{XY}{B}K \qquad \ldots \quad (4)$$

Here V_1 is proportional to X and $K\,\frac{XY}{B}$ is the product term. Thus by adding a constant fraction $\left(1-\frac{D}{B}\right)K$ of X in proper phase with V_2, the product $\mp\frac{XY}{B}K$ is obtained. The scale factor of the multiplier is $\frac{K}{B}$.

From the right hand side of equations (2) and (3), it is obvious that the product term $\frac{XY}{B}K$ is obtained because V_p and V_n contains terms proportional to $(\pm Y-X)^2$ and $(\pm Y+X)^2$ respectively. Evidently the output from the positive half or negative half of the dead-band element can be used for the generation of a square-law function.

The Multiplier Circuit

Figure 2(*a*) shows the general circuit arrangement of the multiplier in schematic form. The transmission characteristics of the dead-band elements N_1 and N_2 are shown in Fig. 2(*b*) and (*c*) respectively. The output from the dead-band

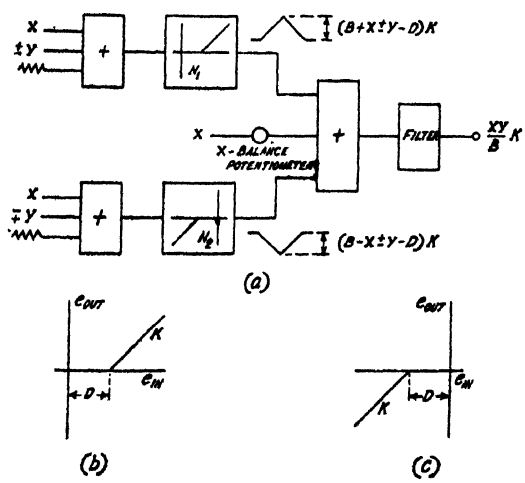

Fig. 2. (*a*) Block diagram of the multiplier.
(*b*) Transmission characteristics of N_1. (*c*) Transmission characteristics of N_2.

element N_1 will be obtained only when input signal is more positive than D. Therefore if B be the amplitude of the triangular wave, peak output will be equal to $(B \pm Y + X - D)K$. Similarly, the output from the dead-band element N_2 will be obtained only when the input signal is more negative than D. The magnitude of the peak output from N_2 will be $(B \pm Y - X - D)K$. These two outputs along with a fraction $-(1-D/B)$ of X is fed into a low-pass filter. The product term is obtained from the output of the filter.

Dead-band Circuits

Figures 3(a) and (b) show the negative and positive halves of the conventional dead-band circuit (Jackson). A is a high gain drift stabilised d.c. amplifier. Voltages $+V_1$ and $-V_2$ are taken from stabilised sources and connected to the input terminals of resistances R_{V_1} and R_{V_2} respectively. The input signals X, $\pm Y$ and the triangular wave are connected to the resistances R_{V_1}, R_{Y_1} and R_{T_1} of the negative half of the dead-band circuit and the signals X, $\pm Y$ and the triangular wave are connected to R_{X_2}, R_{Y_2} and R_{T_2} respectively of the other half of dead-band circuit.

Static Adjustment of the Multiplier

The equation (1) has been derived under assumption that the triangular wave is fed to an ideal dead-band element. This, in other words, means that in the circuit of Figs. 3(a) and (b) if V_1 and V_2 are eaual in magnitude then

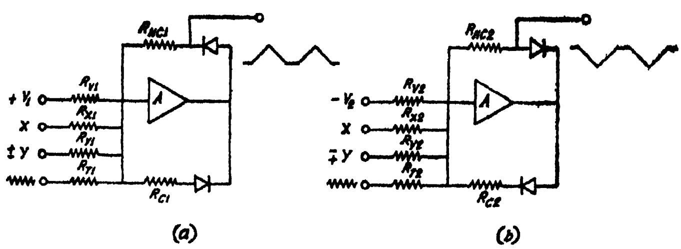

Fig. 3. (*a*) Positive half of the doad-band circuit.
(*b*) Negative half of the dead-band circuit.

Resistances	R_{V_1}	be equal	to	R_{V_2}
	R_{X_1}	,,	,,	,,	R_{X_2}
	R_{Y_1}	,,	,,	,,	R_{Y_2}
	R_{T_1}	,,	,,	,,	R_{T_2}
	R_{nc_1}	,,	,,	,,	R_{nc_2}
	R_{c_1}	,,	,,	,,	R_{c_2}

and the gain of the adding circuit where the outputs from 3(a) and (b) are added should be identical. But in actual practice it is very difficult to adjust so many resistances accurately.

The average value of the output from an adder, when signals from the circuit 3(*a*) and (*b*) are fed to its input points having gains K_a and K_b respectively, is given by

$$E_{out} = K_a \frac{R_{nc1}}{R_{T_1}} \left[\frac{B}{4} - \frac{\left(V_1 + \frac{R_{V_1}}{R_{X_1}} X \pm \frac{R_{V_1}}{R_{Y_1}} Y \right)}{2} \cdot \frac{R_{T_1}}{R_{V_1}} + \frac{\left(V_1 + \frac{R_{V_1}}{R_{X_1}} X \pm \frac{R_{V_1}}{R_{Y_1}} Y \right)^2}{4} \left(\frac{R_{T_1}}{R_{V_1}} \right)^2 \right]$$

$$-K_b \frac{R_{nc2}}{R_{T_2}} \left[\frac{B}{4} - \frac{\left(V_2 - \frac{R_{V_2}}{R_{X_2}} X \pm \frac{R_{V_2}}{R_{Y_2}} Y \right)}{2} \frac{R_{T_2}}{R_{V_2}} + \frac{\left(V_2 - \frac{R_{V_2}}{R_{X_2}} X \pm \frac{R_{V_2}}{R_{Y_2}} Y \right)^2}{4} \left(\frac{R_{T_2}}{R_{V_2}} \right)^2 \right] \quad \ldots (5)$$

Equation (5) can be written in the form,

$$E_{out} = \alpha_0 + \alpha_X X + \alpha_Y Y + \alpha_{XY} XY + \alpha_1 X^2 + \alpha_2 Y^2 \quad \ldots (6)$$

The coefficients α_0, α_X, α_Y, α_{XY}, α_1 and α_2 are related to the circuit parameters. If we adjust the circuit such that the co-efficients α_1 and α_2 are zero then average value of the output is given by

$$E_{out} = \alpha_0' + \alpha_X' X + \alpha_Y' Y + \alpha_{XY} XY$$

where α_0', α_X', α_Y' and α'_{XY} are the coefficients when the circuit has been adjusted for the condition $\alpha_1 = \alpha_2 = 0$. The terms α_0', $\alpha_X' X$ and $\alpha_Y' Y$ can be balanced out at the output by feeding a constant term and terms proportional to X and Y in proper phase at the input of the filter. From expression (5) we find that

$$\alpha_1 \quad \left[\frac{R_{V_1}}{R_{X_1}} \right]^2 \left[\frac{R_{T_1}}{R_{V_1}} \right]^2 \cdot \frac{R_{nc1}}{R_{T_1}} K_a - \left[\frac{R_{V_2}}{R_{X_2}} \right]^2 \left[\frac{R_{T_2}}{R_{V_2}} \right]^2 \frac{R_{nc2}}{R_{T_2}} K_b \quad (7)$$

and

$$\alpha_2 = \left[\frac{R_{V_1}}{R_{Y_1}} \right]^2 \left[\frac{R_{T_1}}{R_{V_1}} \right]^2 \frac{R_{nc1}}{R_{T_1}} K_a - \left[\frac{R_{V_2}}{R_{Y_2}} \right]^2 \left[\frac{R_{T_2}}{R_{V_2}} \right]^2 \frac{R_{nc2}}{R_{T_2}} K_b \quad (8)$$

α_1 can be made zero if

$$\frac{K_a R_{nc_1} R_{T_1}}{R_{X_1}^2} = \frac{K_b R_{nc_2} R_{T_2}}{R_{X_2}^2} \tag{9}$$

and α_2 can be adjusted to zero by satisfying the condition

$$\frac{K_a R_{nc_1} R_{T_1}}{R_{Y_1}^2} = \frac{K_b R_{nc_2} R_{T_2}}{R_{Y_2}^2} \tag{10}$$

The coefficients α_1 and α_2 can be adjusted to zero independently of one another by controlling any two different resistances in the circuit. This method of adjustment has been found to be very convenient in actual practice.

EXPERIMENTAL METHOD OF ADJUSTMENT OF STATIC AND LINEARITY CHARACTERISTICS OF THE MULTIPLIER

The simplified circuit arrangement of the multiplier is shown in Fig. 4. For adjusting the coefficient α_1 of the X^2 term to zero, a low frequency signal is applied to R_{X_1} and R_{X_2} of the dead-band elements and the inputs to the terminals of R_{Y_1}

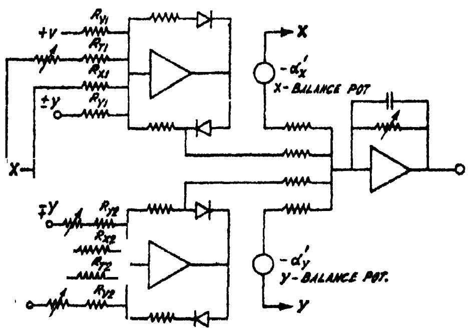

Fig. 4. Arrangement of the multiplier circuit.

and R_{Y_2} are made zero. The X-balance potentiometer is adjusted to make the fundamental component of the output zero. If with this adjustment there is a second harmonic component of the X-signal at the output the variable resistance in series with R_{T_1} is adjusted to minimise the second harmonic component. With the variation of R_{T_1} there will appear some fundamental component at the output. This fundamental component is adjusted to zero for each adjustment of R_{T_1}. By simultaneous adjustment of R_{T_1} and X-balance potentiometer the output from the system is brought below the minimum detection level. Now if the X-signal is varied between zero and full value, the output should remain below the minimum detection level. Any variation in the output with the variation of X-signal indicates the departure from linearity of the X channel. The departure will be mainly due to the nonlinearity of the triangular wave.

TABLE

$X = 45$ volts peak. $Y = 45$ volts d.c.

X-balance potentiometer reading for balancing the product output

X \ Y	1.0	.9	.8	.7	-6	.5	.4	.3	.2	.1	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
.1	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.2	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.3	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.4	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.5	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.6	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.7	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.8	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
.9	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
1.0	.498	.5181	.539	.560	.581	.602	.6225	.6431	.6642	.685	.7057	.7261	.7469	.7679	.7882	.809	.8293	.850	.8705	.891	.911
	For positive values of Y											For negative values of Y									

The coefficient α_2 associated with Y^2 term in expression (6) is similarly adjusted to zero by feeding an a.c. signal at the Y inputs and simultaneously adjusting the Y-balance potentiometer and R_{Y_1}. After α_2 has been adjusted to zero at a particular frequency, and output from the system with the variation of Y-signal frequency will indicate that the phase difference between the two Y input signals is different from 180°. In the circuit designed, the variation with the adjustment in X or Y could easily be kept below 2 millivolts.

For measuring the linearity characteristics of the multiplier a low frequency signal is applied to the X channel and a d.c. signal is applied into Y channel. The Y-signal is adjusted to different values. For each adjustment of Y inputs, the output is balanced out by adjusting the X balance potentiometer. For fixed values of Y input the fraction of X-input required to balance the output is constant for all values of X input for a perfectly linear multiplier. The Table I below gives the reading of the X-balance potentiometer for different values of Y signal for the multiplier designed on the basis of this principle. The linearity characteristic is not given in the conventional four quadrant figure. Since it has been found that with ± 2 mv accuracy of the measuring instrument, there is negligible amount deviation from its linearity characteristics. Hence the linearity characteristic is given by potentiometer readings, required for balancing the product output.

Dynamic Characteristics of the Multiplier

If a d.c. signal is applied into one of the two inputs and a sine wave to the other input, the bandwidth (3db. point of the output sine wave) of the multiplier using 1 Kc/s triangular wave is found to be 30 c/s. Further studies are being made to increase the bandwidth of this type of multipliers.

CONCLUSION

The operation of the multiplier depends upon the square-law functions generated in the two halves of the dead-band element. These square-law function generators using the dead-band element with a triangular wave require least number of components and their static and dynamic accuracies will be much superior to those of biased diode type of square-law function generators. Obviously the static and dynamic accuracies of the multiplier are better. The linearity of the multiplier, particularly in the lower signal range is good. The bandwidth of the multiplier can be increased by using very high frequency triangular wave and using improved filters.

The multiplier is very easy to construct. In a standard analogue computer laboratory, having a single master triangular wave generator, a series of such multipliers can be constructed with usual analogue computer elements.

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